Microeconomic Foundations I

Choice and Competitive Markets

# Student's Guide

# Chapter 4: Revealed Preference and Afriat's Theorem

## Summary of the Chapter

Chapter 4 concerns revealed preference. If we see a finite amount of data concerning the demand of a consumer, what can we tell about the consumer's preferences? In particular:

1. Are these data consistent with our model of a utility-maximizing consumer?
2. What patterns in the data can we expect to see? For instance, does demand for a commodity always rise when the price of that commodity falls (everything else held equal)?

Afriat's theorem, given in the text as Proposition 4.3, answers the first question, under the maintained hypothesis that the consumer in question has locally insatiable preferences. (Without some means of deducing from the data that the consumer strictly prefers one bundle to another, the answer to question 1 is always Yes: Any budget-feasible choices are consistent with utility maximization, namely by a consumer who is indifferent among all bundles. The maintained hypothesis of local insatiability is one way to "see" strict preferences in the data.) Since Afriat's theorem gives necessary and sufficient conditions for a finite set of demand data to be consistent with utility maximization, it can then be used to answer question 2; we can conceivably see any pattern in the data that is not precluded by Afriat's theorem. In particular, so-called *Giffen goods*, a good the demand for which rises with increases in the price of the good, are possible, at least in theory.

Two specific points made in the chapter merit special attention.

1. Concerning Afriat's theorem, the basic condition (no revealed preference cycle with at least one link a revelation of strict preference) is *necessary* if the data are generated by a locally insatiable, utility-maximizing consumer and is *sufficient* to guarantee that the data could come from a consumer with continuous, strictly increasing, and convex preferences. Hence the latter three conditions are empirically untestable from finite *market demand* data, except by throwing out the entire model of utility maximization with local insatiability. (And if you want to understand why I emphasized "market demand," see the text.)
2. Because of income effects, little in the way of comparative static conclusions can come from market (Marshallian) demand data with a fixed level of income. But comparative static conclusions can arise by dealing with compensated demand, where income levels are changed to compensate for changes in prices.

Chapter 4 is short and without much in the way of complications, except for the proof of Afriat's theorem. Your instructor will tell you how much attention to pay to the proof; it's an aesthetically beautiful proof, but not one whose techniques are used elsewhere in the book.

## Solutions to Starred Problems

■ 4.1. For each bundle and each price vector, we compute the cost of the bundle at those prices:

| consumption bundles | price vectors (1,1,1) | (3,1,1) | (1,2,2) | (1,1,2) |
|---|---|---|---|---|
| (10,5,5) | 20 | 40 | 30 | 25 |
| (3,5,6) | 14 | 20 | 25 | 20 |
| (13,3,3) | 19 | 45 | 25 | 22 |
| (15,3,1) | 19 | 49 | 23 | 20 |

We verify first that, in each case, the chosen bundle exhausts the budget constraint. And we conclude from the first set of prices that (10,5,5) strictly dominates the other three. At the second set of prices (3,1,1), none of the other bundles are affordable, so this column generates no restrictions. The third set of prices reveals that $(13,3,3) \succ (15,3,1)$ and $(13,3,3) \succeq (3,5,6)$. And the fourth and final set of prices tells us (only) that $(15,3,1) \succeq (3,5,6)$. This gives no cycles that I can find and, indeed, the data are consistent with $(10,5,5) \succ (13,3,3) \succ (15,3,1) \succ (3,5,6)$. So Afriat's theorem tells us that these data could come from a locally insatiable, utility-maximizing consumer (with continuous, convex, and strictly increasing preferences).

■ 4.4. Since preferences are locally insatiable, $p \cdot x = y$ and $p' \cdot x' = p' \cdot x$. Since $x'$ is chosen when prices are $p'$ and income is $p' \cdot x$, $x' \succeq x$ must hold (since $x$ is affordable). But then $p \cdot x' \geq y = p \cdot x$ must be true; otherwise, local insatiability would tell us that something strictly better than $x'$ is affordable at prices $p$ and income $y$, and such a bundle would be strictly preferred to $x$ by transitivity.

Therefore, $p \cdot x' \geq p \cdot x$ and $p' \cdot x' = p' \cdot x$. But

$$(p' - p) \cdot (x' - x) = p' \cdot x' - p' \cdot x - p \cdot x' + p \cdot x \leq 0,$$

since the first two terms give 0 and the second two are less than or equal to 0.