Microeconomic Foundations I: Choice and Competitive Markets

Student's Guide

Chapter 7: Dynamic Choice

Summary of the Chapter

The theme of the first six chapters of the book has largely been choice by individuals, where the choices have been framed atemporally: The individual has a single choice to make; how do we model that? But most choices, and in particular most of the most important economic choices that individuals make, are part of a sequence of choices. So it is natural to ask (and try to answer) whether and how we adapt the models of choice we have developed to dynamic-choice contexts.

In most of the literature of economics, the answer is presumed to be so obvious that it is never explicitly discussed: An individual in a dynamic-choice situation thinks from the outset about all the choices she has to make and what will be the consequences for herself of the different *strategies* she might employ. She rates each strategy as being just as good as the outcome it engenders. And she chooses to employ, and proceeds flawlessly to implement, an *optimal* strategy; a strategy that leads to the best outcome available to her.

If this answer is accepted, then the issue becomes one of applying the model: Given a relatively complex dynamic-choice problem, how does one find an optimal strategy? The mathematical techniques of dynamic programming are generally employed in the economics literature; Appendix 6 provides you with a primer on these techniques.

That, for the most part, is the "story" about dynamic choice in the mainstream economic literature. The story is not subjected to critical examination. Chapter 7 relates the basics of the standard story and then suggests that critical examination may be in order. The standard story is predicated on (at least) three implicit assumptions:

1. The decision maker has (and acts on) fixed, atemporal preferences over outcomes. She will not change her mind about what she wants.

2. From the outset, the decision maker has full "strategic awareness" of what options she has and will have and how her decisions will translate into outcomes. All this is allowed to depend on the resolution of uncertain events. But unforeseen contingencies, previously unappreciated options, and the general law of unintended consequences are not (relevant) parts of her vocabulary and, in particular, she makes no allowance today for any such possibilities affecting her in the future.
3. The decision maker has the cognitive and computational skills required to evaluate all the strategies she has available or, at least, to find one that is optimal.

Having made these assumptions explicit, the chapter suggests some possible alternatives to the standard story, if the assumptions are violated.

1. If the decision maker's tastes may change through time, she recognizes this, and she acts today in a way that tries to optimize the outcome from the perspective of today's tastes, then she may make choices today that eliminate from her later opportunity sets choices that, today, she would rather she didn't take later.
2. On the other hand, if she recognizes the possibility of unforeseen contingencies, and so forth, she may devote resources today to leaving herself with the flexibility later on to deal with them, even if today she doesn't foresee what they will be.
3. And faced with a truly complex dynamic decision problem, one that is intractable even with the technology of dynamic programming, she may resort to rules of thumb and decision heuristics, ranging from very simple forms of adaptive satisficing (take the first option that appears, that appears to be "good enough") to employing complex but mis-specified statistical models of her environment, to try to understand that environment.

While there are literatures on each of these alternatives, the literatures are small and under-developed. Following this chapter (certainly for the remainder of this volume), we employ the standard, strategic apporach to dynamic decision making. But as an informed consumer of economic methodology, you should (at least) know (1) that, notwithstanding the impression left in much of the literature that this approach is obvious, this approach is based on some pretty heroic assumptions, and (2) it is entirely possible and, I believe, desirable to explore alternatives.

Two sorts of problems can be connected to this chapter. The first sort provides you with practice solving dynamic choice problems of the standard sort, using the techniques of dynamic programming as described in Appendix 6. You should certainly do some problems of this sort; some are given in Appendix 6, with solutions provided in another "chapter" posted at this *Student's Guide*.

The other sort of problem explores alternatives to the standard model, and that's what you will find in the four problems at the end of Chapter 7: Problems 7.1, 7.2, and 7.3 concern the "changing tastes" model, while 7.4 concerns "preference for flexibility." These problems do not build skills that you need for what follows in the book, and you shouldn't bother with them if you aren't curious about alternatives to the standard, strategic model. But if you are curious, here are solutions of the first three. (The text gives you a reference in which you can find the solution to the fourth problem).

Solutions to Starred Problems

■ 7.1. (a) The utility function is strictly concave, so there is a unique solution. The problem is completely stationary, so at the solution, $a_1 = a_2 = b_1 = b_2 = 25$. The utility obtained is $12.875\ldots$

(b) She will choose $a_1 = b_1 = 25$ and $s_1 = 50$ at the first stage, and $a_2 = b_2 = 25$ at the second stage.

(c) The problem here is to maximize $\ln(a_1) + (1 + a_1/25 \ln(a_2) + \ln(b_1) + \ln(b_2)$ subject to the constraint $a_1 + a_2 + b_1 + b_2 \leq 100$ and all variables nonnegative. I solved this numerically (using Excel and Solver) and got (to three decimal places) $a_1 = 61.876, a_2 = 24.179, b_1 = b_2 = 6.963$. Because her tastes are not changing (but are simply nonseparable), it doesn't matter whether she chooses all at once or in two stages; if she has to save, she'll save (roughly) 13.962 at the first stage and wind up with the outcome given in the previous sentence.

(d) If the decision maker doesn't anticipate the shift in her tastes, she'll begin (naively) thinking she is in the setting of part b, consuming $a_1 = b_1 = 25$ and $s_1 = 50$. But then, when she gets to the second stage, she'll have her savings of 50 and want to maximize $\ln(b_2) + 2\ln(a_2)$, where the coefficient 2 comes from the formula $1 + a_1/25$ and the fact that $a_1 = 25$. We can work this out analytically, getting $b_2 = 33.333\ldots$ and $a_2 = 16.666\ldots$. Just for reference sake, let me evaluate the outome $(a_1, a_2, b_1, b_2) = (25, 33.333, 25, 16.666)$ with the first-stage utility function: it gives (to three decimal places) 12.758.

(e) A more sophisticated decision maker with tastes that change in this fashion reasons as follows: "If I consume a_1 and b_1 in the first period, saving $s_1 = 100 - a_1 - b_1$, then my second-period self will choose to maximize $(1 + a_1/25)\ln(a_2) + \ln(b_2)$ subject to the constraint that $a_2 + b_2 \leq s_1$. The solution she will find is

$$a_2 = \frac{25 + a_1}{50 + a_1} s_1 \quad \text{and} \quad b_2 = \frac{25}{50 + a_1} s_1.$$

So I want to choose a_1, b_1, and s_1 to maximize

$$\ln(a_1) + \ln(b_1) + \ln\left(\frac{25 + a_1}{50 + a_1} s_1\right) + \ln\left(\frac{25}{50 + a_1} s_1\right),$$

subject to the constraint that $s_1 + a_1 + b_1 \leq 100$. Using numerical methods, the solution (to three decimal places) is $a_1 = 22.189, b_1 = 25.937$, and $s_1 = 51.874$, with the second-stage choices then being $a_2 = 33.909$ and $b_2 = 17.965$, for a utility of 12.767. What is interesting here is that savings *increases*, which would seem to fly in the face of the discussion in the text, which is all about constraining future selves: Doesn't more savings give more freedom of choice to one's future self? The reason savings increases, though, is because it is clear that the second-stage self will choose a larger than desireable (by the first-period self) a_2 and a smaller than desireable b_2, and to diminish the deleterious effects of the second half of this, the first-period self saves more. But she also cuts back on her first-period consumption of asparagus, to reduce the impact of asparagus addiction.

■ 7.2(a) The problem is to maximize $c_0^{1/2} + 0.5c_1^{1/2} + 0.4c_2^{1/2}$ subject to $c_0 + c_1 + c_2 \leq 100$, for which the answer is $c_0 = 70.922, c_1 = 17.731, c_2 = 11.347$ (each of these approximately, to three decimal places), giving utility level 11.8743421.

(b) Being naive, the individual chooses the same c_0 as in part a, which means she saves $s_0 = 29.078$. But then, at time $t = 1$, she divides this between time $t = 1$ and $t = 2$ consumption in the way that maximizes $c_1^{1/2} + 0.5c_2^{1/2}$, which is, in a 4 to 1 ratio, or $c_1 = 23.262$ and $c_2 = 5.816$, for a $(t = 0)$ uility level 11.7976977.

(c) Being sophisticated, the individual knows that if she saves s_0 at time $t = 0$, at time $t = 1$ she'll consume 80% of that amount and leave 20% for time $t = 1$ consumption. So she picks c_0 (and $s_0 = 100 - c_0$) with that in mind. That is, she picks c_0 to maximize

$$c_0^{1/2} + 0.5[(0.8(100 - c_0))^{1/2}] + 0.4[(0.2)(100 - c_0))^{1/2}],$$

which gives $c_0 = 71.839$, $c_1 = 22.529$, and $c_2 = 5.632$ (as always, rounded to three decimal places), for a $(t = 0)$ utility level 11.798305. That's not much of an increase in her utility over part b, but it's there. Compared with problem 7.1, in this case sophistication leads her to consume *more* in the first period and to save less.

■ 7.3. Suppose $\dot{\succeq}$, defined on Z, is complete and transitive and satisfies

$$\text{For all } z \text{ and } z', \text{ either } z \dot{\sim} z \cup z' \text{ or } z' \dot{\sim} z \cup z', \qquad (G7.2.1)$$

and is antisymmetric when restricted to singleton sets. Define $\succeq_1$ and $\succeq_2$ on X as in the statement of the problem:

$$x \succeq_1 x' \text{ if } \{x\} \dot{\succeq} \{x'\} \quad \text{and} \quad x \succeq_2 x' \text{ if } \{x\} \dot{\sim} \{x, x'\}.$$

Since $\succeq_1$ is, essentially, $\dot{\succeq}$ restricted to singleton sets, it is evident that $\succeq_1$ is complete, transitive, and anti-symmetric.

As for $\succeq_2$, take any pair x and x' from X. By G7.2.1, either $\{x\} \dot{\sim} \{x, x'\} = \{x\} \cup \{x'\}$ or $\{x'\} \dot{\sim} \{x, x'\}$. Hence, by the definition of $\succeq_2$, $x \succeq_2 x'$ in the first case and $x' \succeq_2$ in the second case, therefore $\succeq_2$ is complete. Although it is redundant to say so, note that if $x = x'$, then $\{x\} = \{x, x'\}$, so $\{x\}\dot{\sim}\{x, x'\}$, so $x \succeq_2 x' = x$. On the other hand (and this part isn't redundant) if $x \neq x'$, then by then anti-symmetry of $\dot{\succeq}$, if $\{x\}\dot{\succeq}\{x'\}$, it cannot be the case that $\{x'\}\dot{\succeq}\{x\}$, and so $\{x\}\dot{\succ}\{x'\}$. We know that either $\{x\}\dot{\sim}\{x, x'\}$ or $\{x'\}\dot{\sim}\{x, x'\}$, but since (for $x \neq x'$) $\{x\}\dot{\sim}\{x'\}$ is not possible, we know that *exactly* one of these can be true, and $\succeq_2$ is anti-symmetric. (We'll show that $\succeq_2$ is transitive at the end.)

I assert that

$$\text{For every } z \in Z\text{, there exists some } x \in z \text{ such that } \{x\}\dot{\sim}z. \qquad (G7.2.2)$$

This is shown using induction on the size of z. (The notation $\#z$ will be used for the cardinality of z.) This is clearly true for all singleton sets z, so suppose inductively that it is true for all sets of size $n-1$ and less. Suppose $\#z = n > 1$, and let x^* be any element of z and $z^0 = z \setminus \{x^*\}$. Then z^0 has cardinality $n-1$, and by the induction hypothesis, for some $x^0 \in z^0$, $\{x^0\}\dot{\sim}z^0$. But by G7.2.1, either

$$z^0\dot{\sim}z^0 \cup \{x^*\} = z \quad \text{or} \quad \{x^*\}\dot{\sim}z^0 \cup \{x^*\} = z.$$

In the second case, $z\dot{\sim}\{x^*\}$, while in the first case, $\{x^0\}\dot{\sim}z^0\dot{\sim}z$, hence $\{x^0\}\dot{\sim}z$. Since one of these two must hold, we know that z is indifferent to the singleton set consisting of one of its elements, completing the induction step and the proof by induction.

I assert that

$$\text{If } \{x\}\dot{\sim}z\text{, for } x \in z\text{, then } \{x\}\dot{\sim}z' \text{ for all sets } z' \text{ such that } x \in z' \subseteq z. \qquad (G7.2.3)$$

The proof is again by induction on the size of z. This is vacuously true if z is a singleton set. Suppose it is true for all sets with $n-1$ or fewer elements, and let z be a set of n elements for $n > 1$. Suppose $\{x\}\dot{\sim}z$ for some $x \in z$. Let z' be any subset of z with $n-1$ elements, one of which is x, and let x' be the element of z left out of z'. Either $\{x'\}\dot{\sim}z$ or $z'\dot{\sim}z$ by G7.2.1. In the latter case, since $\{x\}\dot{\sim}z$, we conclude that $\{x\}\dot{\sim}z'$. In the former case, transitivity of $\dot{\sim}$ implies that $\{x\}\dot{\sim}\{x'\}$, contradicting anti-symmetry of $\dot{\succeq}$ restricted to singleton sets. So $\{x\}\dot{\sim}z'$. But by the induction hypothesis, this implies that $\{x\}\dot{\sim}z''$ for all subsets of z' that contain x. And since this is so for all subsets z' of z with $n-1$ elements, we have the induction step and, by induction, the asserted property.

Now we will show that $\succeq_2$ is transitive. Suppose $x \succeq_2 x'$ and $x' \succeq_2 x''$, which means that $\{x\}\dot{\sim}\{x, x'\}$ and $\{x'\}\dot{\sim}\{x'x''\}$. If $x = x' \neq x''$, then $x \succeq_2 x''$ is obvious, and similarly in the case $x \neq x' = x''$. If $x = x' = x''$, transitivity is clear. And the case $x \neq x'$ but $x = x''$ is impossible by anti-symmetry. So we are left with the case where these three elements are distinct from one another. I assert that $\{x\}\dot{\sim}\{x, x', x''\}$. We know that if this is not true, then either $\{x'\}\dot{\sim}\{x, x', x''\}$, or $\{x''\}\dot{\sim}\{x, x', x''\}$. In the first case, we would then know that $\{x'\}\dot{\sim}\{x,x'\}$, contradicting anti-symmetry. In the second case, we would know that $\{x''\}\dot{\sim}\{x',x''\}$, contradicting anti-symmetry. Hence it must be that $\{x\}\dot{\sim}\{x,x',x''\}$. But then $\{x\}\dot{\sim}\{x,x''\}$ by G7.2.3, and therefore $x \succeq_2 x''$.

G7.2.2 tells us that, for each $z \in Z$, some $x \in z$ satisfies $\{x\}\dot{\sim}z$. Since $\dot{\succeq}$ is antisymmetric on singleton sets, this x, which we henceforth label $x_2(z)$, is unique. Take any $x \neq x_2(z)$ from z. By G7.2.3, $x_2(z)\dot{\sim}\{x_2(z),x\}$ so, by definition, $x_2(z) \succeq_2 x$. Of course, $x_2(z) \succeq_2 x_2(z)$, and so

$$\text{For all } z,\ x_2(z) \succeq_2 x \text{ for all } x \in z. \tag{G7.2.4}$$

Finally, by simple transitivity properties of $\dot{\succeq}$, for all z and z', $z\dot{\succeq}z'$ if and only if $\{x_2(z)\}\dot{\succeq}\{x_2(z')\}$ if and only if $x_2(z) \succeq_1 x_2(z')$. But this tells us that $\dot{\succeq}$ on Z is "explained" by the changing-tastes model, where first-stage preferences over meals are given by $\succeq_1$ and second-stage preferences over meals are given by $\succeq_2$.

As mentioned in the text, this proof, although perhaps a bit tedious, is greatly helped along by the assumption that $\dot{\succeq}$ is anti-symmetric when reduced to singleton sets (which, in words, is the assumption that the decision maker is not indifferent between any pair of distinct meals). If you wish to see the proof for the case where this assumption is dropped, go to Gul and Pesendorfer (2005).