Microeconomic Foundations I:
Choice and Competitive Markets

Student's Guide

Chapter 12: Producer and Consumer Surplus

Summary of the Chapter

To this point, and with the exception of Chapter 8, this volume has concerned the behaviors of single consumers and single firms. The environment in which these consumers and firms act might contain other consumers and/or firms, but the interactions between diverse consumers and firms have not been an issue. That begins to change in this chapter.

A natural first step is to ask, Is the market behavior of a collection of firms or a collection of consumers similar to the behavior of single firms and consumers? That question, in microeconomics, goes by the name *aggregation*, and is the topic of next chapter. Anticipating a bit, we'll discover that aggregation works well for firms, but is problematic for consumers. Nonetheless, to make use of the models we've developed, it is helpful to have ways to "aggregate," and in this chapter we warm up to the harder developments of next chapter by asking, (How) Can we aggregate the impact on firms or consumers of a change in, say, the price of a single good?

Most readers will know the practical answer to this question: Economists measure the aggregate impact on firms of a change in the price of a good by the change in *producer surplus*, while the impact on consumers is measured by the change in *consumer surplus*. In each case, "impact" is measured on a scale of dollars (or whatever price numeraire is in use), which allows us to add up the impacts on individual firms and/or consumers, to get a measure of aggregate impact.

But what is the theoretical rationale for producer and consumer surplus? That is the specific question asked and answered in this chapter.

For firms, the answer is simple and straightforward. If (say) the price of commodity i changes, we can use the result that

$$\frac{\partial \pi}{\partial p_i} = z_i^*$$

(where π is the profit function of the firm and for prices where z_i^* is unique, which we show is "most" prices) to conclude that the change in producer surplus for a single firm is equal to the change in its profit owing to the price change. This aggregates very nicely.

But for consumers, the story is not so simple. Owing to income effects in Marshallian demand, there is no single dollar-denominated measure of "the impact on the consumer of a change in the price of a commodity." Two approximate measures, the so-called *compensating* and *equivalent variations*, are used by economists. These two measures can be expressed as changes in the value of the consumer's expenditure function (from Chapter 10) over the range of the changed prices. And we know from Chapter 10 that

$$\frac{\partial e}{\partial p_i} = h_i,$$

where e is the expenditure function for the consumer and h_i is Hicksian demand for commodity i and for prices where Hicksian demand is single valued (for commodity i), which we show is most of them. Hence the integral "under" the appropriate Hicksian demand function—consumer surplus based on Hicksian demand—can be used to compute the compensating and equivalent variations. And we conclude by giving conditions under which traditionally computed consumer surplus—the integral "under" Marshallian demand—must lie between these two variations.

Solutions to Starred Problems

■ 12.1. This is straightforward. We know that every full solution to the firm's profit-maximization problem at a given price vector p is a subgradient of the convex profit function π. Suppose z and z' were two solutions at p with $z_1 \neq z_1'$. Being a subgradient means that $p \cdot z = \pi(p)$ and $p' \cdot z \leq \pi(p')$ for all $p' \neq p$. Fixing all prices other than the first, this means that $p_1 z_1 + \sum_{i=2}^k p_i z_i = \pi(p)$ and $p_1' z_1 + \sum_{i=2}^k p_i z_i \leq \pi(p_1', p_2, \ldots, p_k)$ for all $p_1' \neq p_1$. That is, z_1 is a subgradient of the function $p_1' \rightarrow \pi(p_1', p_2, \ldots, p_k)$. And the same is true of z_1'.

But since π is convex overall, it is a convex function of p_1' alone; if it is differentiable in p_1, it has a unique subgradient at p_1, namely its derivative at p_1. So it is not possible that $z_1 \neq z_1'$; moreover, the sole element of $Z_1^{f*}(p_1^0)$ is indeed the derivative of π in its first argument at p_1.

■ 12.4. A picture (with its legend) suffices: See Figure G12.1.

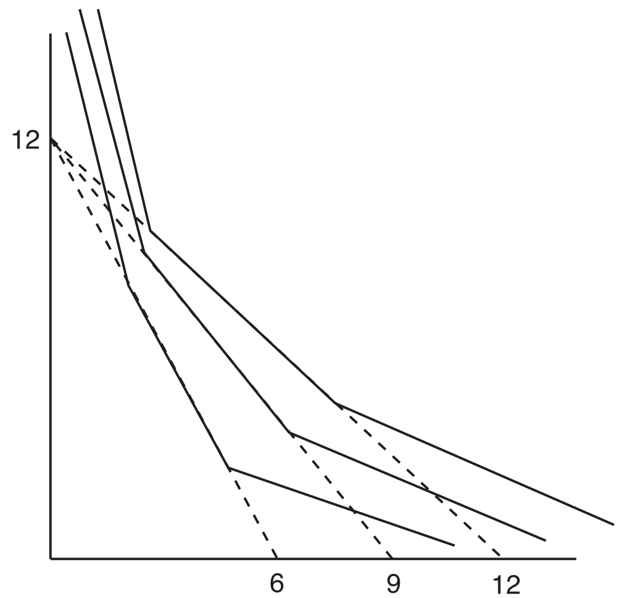

Figure G12.1. Solution to Problem 12.4. The figure shows indifference curves for a consumer in the case of two goods. To construct the figure, we first construct dashed lines that hit the x-axis at different values and the y-axis at $(0, 12)$. Then we construct indifference curves (the solid lines): They are parallel to one-another "above" the dashed lines, but run along the dashed lines (so are not parallel) each for an interval of values. Now imagine a consumer with these indifferent curves, with $y = 12$, with the price of the y good equal to 1, and for various prices of the x good. If the price of the x good is 1, the consumer chooses any of the points that lie along the highest indifference curve shown, where that curve lies along the dashed line; if the prices of the x good is 4/3, she chooses any of the points along the second indifference curve shown, where that curve hits "its" dashed line; if the price of the x good is 2, she chooses any of the points along the third indifference curve, where that curve hits "its" dashed line. The point is that for each price of the x good, the optimal level of Marshallian demand for that good consists of an interval of points. Of course, this is "exceptional": If the ratio of y to the price of the y good is anything other than 12 to 1, this behavior is not observed. But the point is that this sort of exceptional case is, in theory, possible.