Errata in the Text

Here is a list of errata found (so far) in the text. Please note: Princeton University Press is kindly allowing me to correct a number of these in later printings of the book. Assuming you have a physical copy of the book, look at page 122. If there is a Problem 5.7 on this page, you hold a copy of the book with a number of corrections made; if there is no Problem 5.7, you have an early printing, which does not have these corrections.[1] Below, I list corrections as follows: First are major flaws that were found after the corrections were made (and so are germane to all readers); then are minor flaws found after the corrections were made; then major flaws that appear (only) in early printings; and finally minor flaws found in early printings. (For the moment, there are no entries in the first two categories. But I'm relatively sure that some will appear.)

If you happen to find a typo (or something worse) that is not included in this list, I'd be grateful if you would email it to me, at kreps@stanford.edu.

References to line numbers mean lines of text, including any displays as a single line and any headers as a line. A reference, say, to line -10 means line 10 from the bottom.

I interpret the term "flaw" broadly, here, including errors of omission as well as of commission.

If you have an ebook version of the text (for instance, a Kindle edition): I know of at least one typo in the Kindle edition that is not in the paper version of the text; I assume there are others. I will not attempt to keep a record of all such "extra" typos here, but if you have an ebook version and you believe you have found a typo not on this list (or if you are just confused by what you see), you should check with a paper version of the book. Sorry for the added work, but as I write this, the publisher is as mystified as am I about how this could happen.

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Major flaws in all printings

(None so far)

Minor flaws in all printings

Page 44 first full paragraph: This is a flaw, but not easy to fix. In this praragraph, I use a lower case u for the utility function, where the argument is (x, m). The use of lower case for the utility function is pretty standard throughout the chapter, so that's why I did it. However, Definition 2.15 uses an upper case U for utility functions over bundles (x, m) and a lower case u for the sub-utility function over x. So, perhaps, in this paragraph, all those lower case u's would be better as upper case U's. (Best of

[1] If you have an ebook version of the book, the same test applies. But see following for a caveat.

all would be to have as the display in Definition $u(x, m) = v(x) + m$, for some sub-utility function v. But that change, made in all the places subsequently that would be required, would probably be more confusing than helpful. So, in this paragraph on page 44, use either u.c. or l.c. as you find more consistent.

Major flaws in early printings, corrected in later printings

There are seven of these, including some absolute howlers:

Page 35, as well as elsewhere in Chapter 2: There is nothing wrong with what the text says. But the text omits a subtle but substantial point. Line 13 proposes the conjecture that *if preferences $\succeq$ are convex, they admit **at least one** concave numerical representation.* The text correctly says that this conjecture is not true and, in fact, Problem 2.8 gives two examples (and challenges you to find an example involving preferences that are both convex and continuous, a much harder task). But there is a counterexample to the conjecture one can give that is a good deal simpler than the two counterexamples in Problem 2.8, namely lexicographic preferences, as defined in Problem 1.10. These preferences (extended, say, to all of R^2_+) are both strictly monotone—easy to see—and convex—maybe a bit harder to see—but they cannot be represented by *any* utility function, let alone one that is concave or strictly increasing.

This illustrates a larger issue of omission in the chapter. Contrast the two statements *If preferences $\succeq$ have property X, then **every** utility function u that represents $\succeq$ has property Y* and *If preferences $\succeq$ have property X, then **some** utility function u that represents $\succeq$ has property Y*. It is somewhat natural to feel—and the chapter may give you the feeling—that the first statement is stronger than the second; that is, whenever the first is true, the second is as well. But if you get that feeling, be careful! The first statement can be true because there is *no* utility function u that represents $\succeq$, hence every u has the property Y by default. But the second implicitly guarantees that $\succeq$ has at least one representation. I believe that, in this regard, the text never says anything that is wrong. But it sure doesn't do justice to this somewhat subtle point.

In later printings, to make this point, I add a part b to Problem 2.1: Consider lexicographic preferences as defined in Problem 1.10. Which of the properties listed in the left-hand column of Table 2.1 are satisfied by lexicographic preferences? How do your answers to this question square with (a) what you answer in the second-to-left column of the table and (b) the undisputed fact that lexicographic preferences admit no numerical (utility) representation? (This is discussed in the *Student's Guide* if you can't figure it out on your own.)

While I'm on this point, let me add: In Table 2.1, the third row from the bottom concerns strict separability. In filling in what strict separability implies in terms of numerical representations, note that the relevant proposition assumes strict separability *and more*. The answer given in the *Student's Guide* assumes the relevant *and more*.

Page 99 Proposition 5.10: The text reads "If preferences ... are continuous in the weak topology ...." Since I'm not defining the topology here, this is at best ambiguous; readers well-trained in mathematics will assert that it somewhere between an abuse of terminology and just wrong. It should say "If preferences ... are continuous in the topology of weak convergence [of probability measures]...," (which is in fact a weak* topology). I fix this in later printings, adding to this chapter Problem 5.7, which asks you to prove the proposition (and which supplies relevant facts about the topology of weak convergence). You can learn about this, including seeing (a sketch of) the proof of the proposition, by going to the final bits of the *Guide* Chapter 5.

Pages 248 to 249: The motivational paragraph that starts on the bottom of page 248 is, at best, misleading. It seems to say that for u to be locally insatiable, e should be strictly increasing. But according to Proposition 10.4 and its proof, e is strictly increasing whether u is locally insatiable or not. Both Proposition 10.3 and, more importantly, the proof of local insatiability inside the proof of Proposition 10.16 (page 251) make clear that local insatiability of u is related to *continuity* of e *jointly* in p and v. So, the paragraph at the bottom of page 248 makes a lot more sense if it reads "Besides this, e should be concave and homogenous of degree 1 in p. It should be nondecreasing in p, strictly increasing in v, and unbounded in v for each p. And, if we want u to be locally insatiable, it should be continuous jointly in p and v."

Page 354 Proposition 14.14: This proposition (as stated in early printings) is quite wrong. It has two problems, one easily fixed (a matter of normalizing prices) and one much less so. Because the issues raised are substantial, I have moved the solution of Problem 14.11 (which asks you to prove the proposition) to the *Student's Guide*. If you go to the solution of this problem in the *Guide* Chapter 14, a lengthy explanation is provided.

Page 468, Proposition A3.28: Pure "think-o." If the Hessian is negative definite, the function is strictly concave. But, as illustrated by the strictly concave function $f(x) = x^4$, the Hessian of a strictly concave function can be (only) negative semi-definite in places. So please, in the proposition, change "strictly concave if and only if" to "strictly concave if," both times the expression appears.

Page 451 bottom: (A very grievous error!) This only works if the a_n are nonnegative, in which case the absolute value signs in the condition that $\sum_n |a_n| < \infty$ are superfluous.

Page 476, especially the statement of Berge's Theorem and footnote 4: I thought that I make enough assumptions to rule out $A(\theta) = \emptyset$, but I haven't. Suppose the domain of θ is R_+, $F(z, \theta) \equiv 0$, and $A(\theta) = \{1/\theta\}$ for $\theta > 0$ and $= \emptyset$ for $\theta = 0$. Of course, $Z(\theta)$ must be identically $A(\theta)$, and if $B(\theta) \equiv A(\theta)$, I believe all the stated assumptions hold. But the conclusions of the theorem do not hold. I see no way around adding the assumption that A is nonempty valued, in which case everything is fine. So that assumption is added in corrected versions, and you should add it here.

Once this correction is made, the proof offered is fine. But students have found the argument given in the proof to be hard to follow, and so I have rewritten the proof in a manner that, I hope, is clearer. Because Berge's theorem is so important to developments in the book, at the very end of this Errata document, I reproduce both the corrected statement of the Theorem and the new (and, I hope, clearer) proof.

Minor flaws in early printings

Page 8 line 17: "abd" should be "and"

Page 20 line 8: "strictly preferred X to Y" should be "strictly preferred Y to X"

Page 35 line 2: "Proposition 1.19" should be "Proposition 1.20"

Page 40 first line of Proposition 2.12: Insert "are" so that it reads "Preferences $\succeq$ are weakly..."

Page 40 display near the bottom of the page: $u(x^K)$ should be $u(x_K)$

Page 54 line 5: b should be p

Page 57 line 23 (the second line in *Step 1***:** The subscript i on μ should be a j.

Page 70 statement of Afriat's Theorem: I remind you (per footnote 2 on page 31) that as the book progresses, I become quite sloppy, using *increasing* for preferences instead of *monotone*. Here is an example (the first?).

Page 92, line -17: "not" should be "now"

Page 97, line 17: The end of the line should read "some function $U : X \to R$..." That is, the domain of U is X and not Z.

Page 118 statement of Problem 5.4: It should read "With regard to the 'run the horses second' discussion beginning on the bottom of page 105..."

Page 142 Problem 6.7: You should assume that x_1 and x_2 are constrained to be non-negative. And, in the middle of the second paragraph, c_1 should be x_1.

Page 182 third full paragraph: "social welfare functionals" should be "social utility functionals," twice.

Page 226, lines 2 to 3: Given the concavity of V, I'm not certain I need this, but to be safe: Put the word "continuously" just before "differentiable," so it reads "...if this (vector-valued) function is continuously differentiable..."

Page 250: (I seem to have been asleep when proof-reading this chapter. Lots of small stuff:) Line 9 should say "...Fix v' and v with $v' < v$..." Two lines further on, "...such the..." should be "...such that..." And, three lines from the bottom of the page, (10.5) should say (10.6). Skipping to page 253, in the third line after the italicized portion, (10.6) should be (10.7). And on page 254, in the statement of Proposition 10.18, y should be in R_+, not R_+^k.

Page 269 line -2 and line -1, and page 270, line 3: In all three cases, the suprema in y

should be over $y \in R_+$ and not $y \in R_+^k$.

Page 270, proof of Proposition 11.5, line 6 of the proof: "…such that $v \geq u(x^i)$…" should be "…such that $v \geq u'(x^i)$…" And four lines further on, the v^0 should be v (no superscript 0).

Page 274 statement of Proposition 11.9: This isn't a typo, but to be clear, the (11.7) in the last line of the Proposition refers to Condition (11.7) and not Proposition 11.7.

Page 325 line 9: The u.c. T should be a l.c. t.

Page 328 line -15: "is it" should be "it is".

Page 334 line 17: u^i at the very start of the line should be u^h.

Page 339, Definition 14.7, 2nd line: it should read $\ell = 1, \ldots, n$ and not $\ldots, N$.

Page 341, 2nd from last line in the proof: $\prod_{\ell=1}^n A_i$ should be $\prod_{\ell=1}^n A_\ell$.

Page 344 first few lines: all p_n should be p^n.

Page 349 3rd line before the endproof mark: change p_j to ζ_j.

Page 405 very last line: The last square bracket in this display is superfluous.

Page 422 very last line: near the end of the line, $p(\zeta^h)$ should be $\boldsymbol{p}(\zeta^h)$.

Page 454, statement of Carathédory's Theorem: Insert "be" between "can" and "written."

Page 475 line 6 in the proof: l.c. y should be an u.c. Y

Page 482 line 3: "pari pasu" should be "pari passu"

Page 489 line 10: techical should be technical

Page 493 line 2: "Kolmogov's" should be "Kolmogorov's."

Correct statement of Berge's Theorem and the new proof:

Proposition A4.7 (Berge's Theorem, also known as the Theorem of the Maximum). *Consider the parametric constrained-maximization problem*

$$\textit{Maximize} \;\; F(z, \theta), \;\; \textit{subject to } z \in A(\theta).$$

Let $Z(\theta)$ be the set of solutions of this problem for the parameter θ, and let $f(\theta) = sup \; \{F(z,\theta); z \in A(\theta)\}$. If

a. F is a continuous function in (z, θ),

b. $\theta \Rightarrow A(\theta)$ is lower semi-continuous and nonempty valued (that is, $A(\theta) \neq \emptyset$ for all θ), and

c. *there exists for each θ a set $B(\theta) \subseteq A(\theta)$ such that $Z(\theta) \subseteq B(\theta)$, $\sup\{F(z,\theta) : z \in B(\theta)\} = \sup\{F(z,\theta) : z \in A(\theta)\}$, and $\theta \Rightarrow B(\theta)$ is an upper semi-continuous and locally bounded correspondence.*

Then:

d. *$Z(\theta)$ is nonempty for all θ, and $\theta \Rightarrow Z(\theta)$ is an upper semi-continuous and locally bounded correspondence; and*

e. *the function $\theta \to f(\theta)$ is continuous.*

Identical conclusions hold if the optimization problem calls for minimizing F rather than maximizing F.

Before giving the proof of Berge's Theorem, we give a corollary that shows why we were interested last section in singleton-valued correspondences.

Corollary A4.8. *In the situation of Proposition A4.7, if in addition you know that $Z(\theta)$ is a singleton set $\{z(\theta)\}$ for all θ in some (relatively) open set of parameter values, then $z(\theta)$ is a continuous function over that set of parameter values.*

Proof of the corollary. Berge's Theorem establishes that the solution correspondence Z is upper semi-continuous and locally bounded. Apply Proposition A4.6. ∎

In standard statements of this theorem, $B(\theta)$ doesn't appear; it is assumed that $A(\theta)$ is continuous. But the more general version of the result given here permits smoother application of the result in some cases encountered in the text.

Proof of Berge's Theorem. Since B is upper semi-continuous and locally bounded, it is compact valued. Since $A(\theta)$ is nonempty valued, $\sup\{F(z,\theta) : z \in A(\theta)\}$ is either finite or $+\infty$; since $\sup\{F(z,\theta) : z \in A(\theta)\} = \sup\{F(z,\theta) : z \in B(\theta)\}$ (condition c), we conclude that $B(\theta)$ is nonempty for each θ. Nonemptiness of $Z(\theta)$ for all θ then follows from Proposition A2.21, because F is continuous in z and $B(\theta)$ is nonempty and compact for each θ. This also implies that $f(\theta) < \infty$ for all θ.[2]

To establish continuity of f, suppose $\{\theta_n\}$ is a sequence of parameter values with limit θ. Let z be any solution to the problem at θ, so that $f(\theta) = F(z,\theta)$. Since A is lower semi-continuous, we can find $z_n \in A(\theta_n)$ for each n such that $\lim_n z_n = z$. But then, by continuity of F, $\lim_n F(z_n,\theta_n) = F(z,\theta) = f(\theta)$. And since $f(\theta_n) \geq F(z_n,\theta_n)$

[2] Suppose $A(\theta) = \emptyset$ for some θ is permitted, with the convention that $\sup\{F(z,\theta) : z \in A(\theta)\} = -\infty$ when $A(\theta) = \emptyset$. In particular, suppose the domain of θ is R_+, $F(z,\theta) \equiv 0$, and $A(\theta) = \{1/\theta\}$ for all $\theta > 0$ and $= \emptyset$ for $\theta = 0$. For $A \equiv B$, do the rest of the conditions (besides assuming that A is nonempty valued) of Berge's Theorem hold? Does the conclusion hold?

for each n, $\liminf_n f(\theta_n) \geq f(\theta)$; f is lower semi-continuous at θ. Hence, continuity of f at θ can fail only if, for some sequence $\{\theta_n\}$ converging to θ and some $\epsilon > 0$, $f(\theta_n) > f(\theta) + \epsilon$ for all n. Suppose such a sequence exists; choose $z_n \in Z(\theta_n)$ (hence $f(\theta_n) = F(z_n, \theta_n)$) for each n. $Z(\theta_n) \subseteq B(\theta_n)$ implies $z_n \in B(\theta_n)$ for each n. Since B is locally bounded and $\theta_n \to \theta$, we can extract from the sequence $\{z_n\}$ a convergent subsequence $\{z_{n'}\}$, and since B is upper semi-continuous, the limit z of this convergent subsequence is in $B(\theta) \subseteq A(\theta)$. But then $f(\theta) \geq F(z, \theta) = \lim_n F(z_n, \theta_n)$ (by continuity of F) $= \lim_n f(\theta_n)$, contradicting the alleged hypothesis. The function f is continuous.

By assumption, B is locally bounded and $Z(\theta) \subseteq B(\theta)$, hence Z is locally bounded. Suppose $\{\theta_n\}$ and $\{z_n\}$ are sequences of parameters and variables such that: $\lim_n \theta_n = \theta$; $\lim_n z_n = z$; and $z_n \in Z(\theta_n)$, so that $f(\theta_n) = F(z_n, \theta_n)$, for each n. Continuity of F (assumed) and f (just proved) then tells us that $F(z, \theta) = \lim_n F(z_n, \theta_n) = \lim_n f(\theta_n) = f(\theta)$. Since $z_n \in Z(\theta_n) \subseteq B(\theta_n)$ and $\theta \Rightarrow B(\theta)$ is upper semi-continuous, $z \in B(\theta) \subseteq A(\theta)$, hence $z \in Z(\theta)$. We conclude that Z is upper semi-continuous and locally bounded. ∎